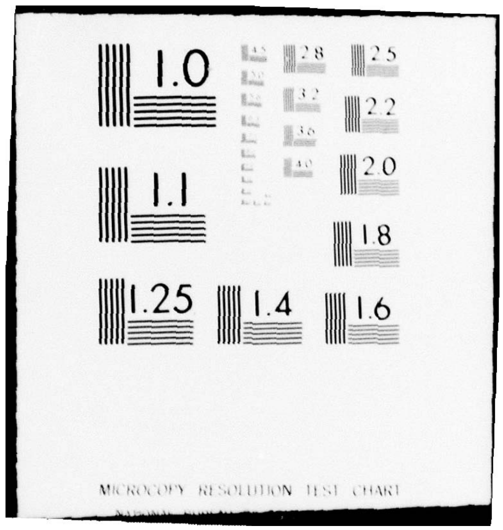
AD-A061 975 METEOROLOGY INTERNATIONAL INC MONTEREY CALIF F/G 20/1
SOUND PROPAGATION IN THE SEA: THE FINITE CHANGE IN WAVEFRONT DI--ETC(U)
JAN 68 M M HOLL N63134-67-M-2854

UNCLASSIFIED NL

1 OF 1

AD
A061975

END
DATE
FILMED
2-79
DDC


1.0
1.1
1.25
1.4
1.6
1.8
2.0
2.2
2.5
2.8
3.2
3.6
4.0
MICROCOPY RESOLUTION TEST CHART

METEOROLOGY INTERNATIONAL INCORPORATED

P. O. BOX 349 • MONTEREY, CALIFORNIA 93940

LEVEL III

1

6

Sound Propagation in the Sea: THE FINITE CHANGE IN WAVEFRONT DIVERGENCE IN PASSING THROUGH DISCONTINUITIES IN SOUND-SPEED GRADIENT AS EFFECTED AT REFLECTIONS.

10

by M. M. Holl

9

Project M-143: Technical Note ~~One~~ no. 1,

11

January 1968

12 9p.

Prepared for:

The Officer-in-Charge
Fleet Numerical Weather Facility
Naval Postgraduate School
Monterey, California

15

Contract No. N63134-67-M-2854

Sound Propagation in the Sea:
THE FINITE CHANGE IN WAVEFRONT DIVERGENCE IN PASSING THROUGH DISCONTINUITIES IN SOUND-SPEED GRADIENT AS EFFECTED AT REFLECTIONS

1. Definitions

We focus our attention on the problem of sound intensity mapping in a vertical plane. For a point source, each selected ray is associated with an angle, γ_S , at which the ray emanates from the source; by convention this angle is measured clockwise from the horizontal. The labelling of rays by their source-emission angles, γ_S , defines γ_S wherever the medium is traversed by the rays. It should be noted that γ_S is not necessarily single valued throughout the medium; more than one ray may pass through a point. Such multiple values are associated with foldings of the wavefront. We may refer to a spread of rays extending from one fold to another as a "family" of rays.

For a family of rays, in the vertical plane, the specific wavefront length, denoted by L , is defined at each point by

$$\frac{\mathbf{n}}{L} \equiv \nabla \gamma_S \tag{1}$$

where $\mathbf{n}$ is a unit vector, clockwise normal to the ray direction at that point. Negative L thus implies γ_S is decreasing in the direction $\mathbf{n}$; sign changes in L are important indications of foldings of the wavefront as at caustics or by reflections.

The differential equation for integrating L along a ray is developed in Reference [1] as

$$\frac{\partial^2 L}{\partial s^2} = -\frac{L}{C}\left[\frac{\partial^2 C}{\partial n^2} - \frac{1}{L}\frac{\partial C}{\partial s}\frac{\partial L}{\partial s}\right] \tag{2}$$

where s is ray path, C is sound speed and n is ray normal. The initial conditions at the source point are

$$L_S = 0 \quad , \quad (\partial L/\partial s)_S = 1 \tag{3}$$

The parameter L may be integrated along each ray beginning at the source. The advantages of such direct integration include (1) the preclusion of the problem of measuring specific spreading by geometric measurement over finite distances between diverse rays, (2) the indication, afforded by sign changes in L, of foldings of the wavefront caused by reflections or at caustics, and (3) the indication, afforded by the spatial continuity in L, of the adequacy of the intensity resolution afforded by the selected family of rays.

Turning to Fig. 1 we note that the reflected rays, after surface reflection at A, have γ_S increasing to the left of the ray direction, *implying negative* L by convention. To accomodate the convention the sign of L must be changed at the point of reflection, by special provision in the integration of Eq. (2) along a ray. At B, L passes through zero, again changing sign; this sign change is inherent in the integration of Eq. (2) along a ray. Such foldings of the wavefront in the medium are called caustics; the caustic is the locus of L = 0 from ray to ray.

Not only is special provision required at reflection to change the sign of L but, generally, also the spreading rate requires special treatment. At reflection the results are such as they would be if the medium were to continue in mirror image beyond the reflecting surface, as shown by the dashed sound-speed profile and the dashed continuation of the rays.

For the sound speed having a gradient normal to the reflecting surface, the ray effectively passes through a kink (gradient discontinuity) in the profile at the point of reflection.

The integration of Eq. (2) along a ray can only accomodate the effect of continuous gradients of the sound speed, C. Thus special treatment is necessary in passing through gradient discontinuities; the spreading rate, $\partial L/\partial s$, abruptly changes. We shall develop the treatment for gradient discontinuities -- interfaces within the medium -- and adapt the treatment to reflection.

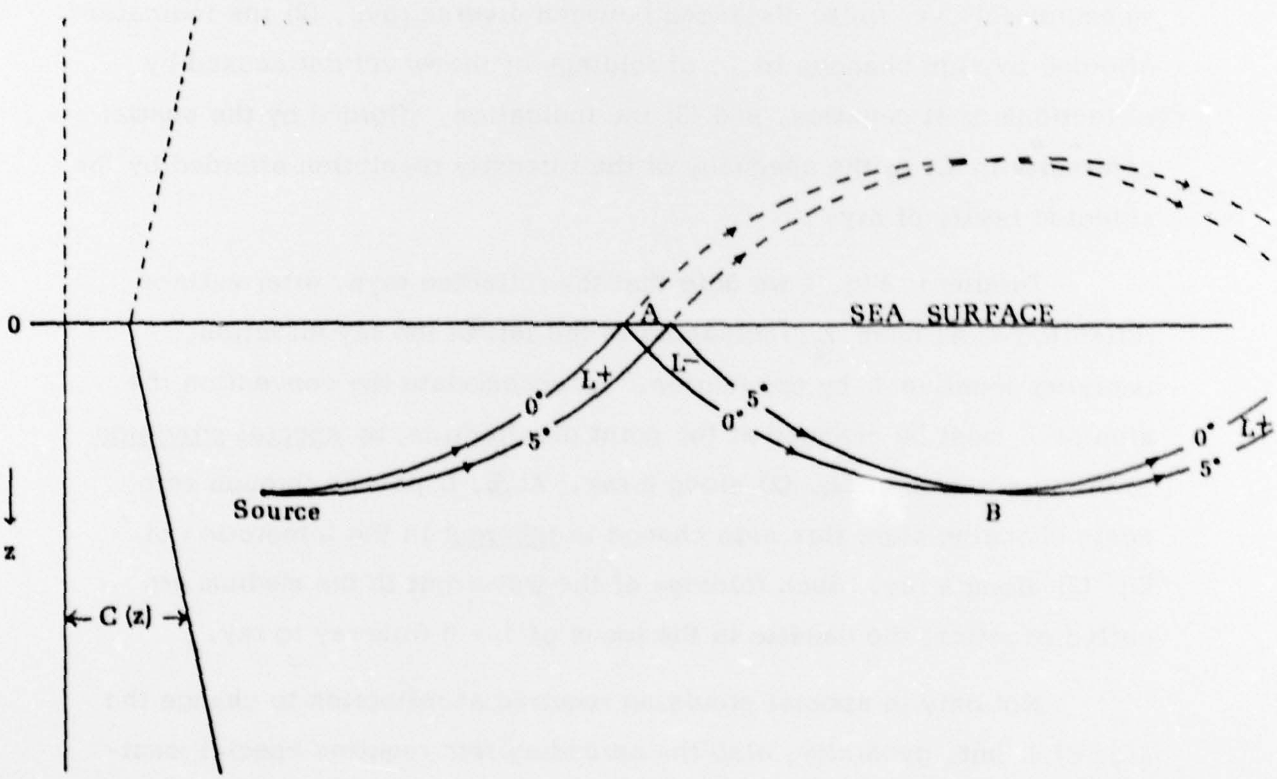


Fig. 1 The traces of two rays, $\gamma_S = 0^\circ$ and $\gamma_S = 5^\circ$, leaving a point source, in a sound speed distribution which increases linearly with depth.

2. Treatment at Interfaces

Depending on the scale of such features of -- and on the resolution of the numerical representation of -- the sound speed distribution, it may be warranted and even desirable to treat strong changes in gradient by an interface of discontinuity.

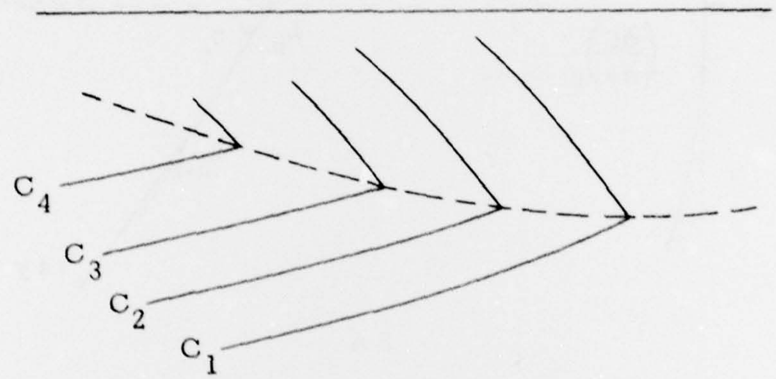
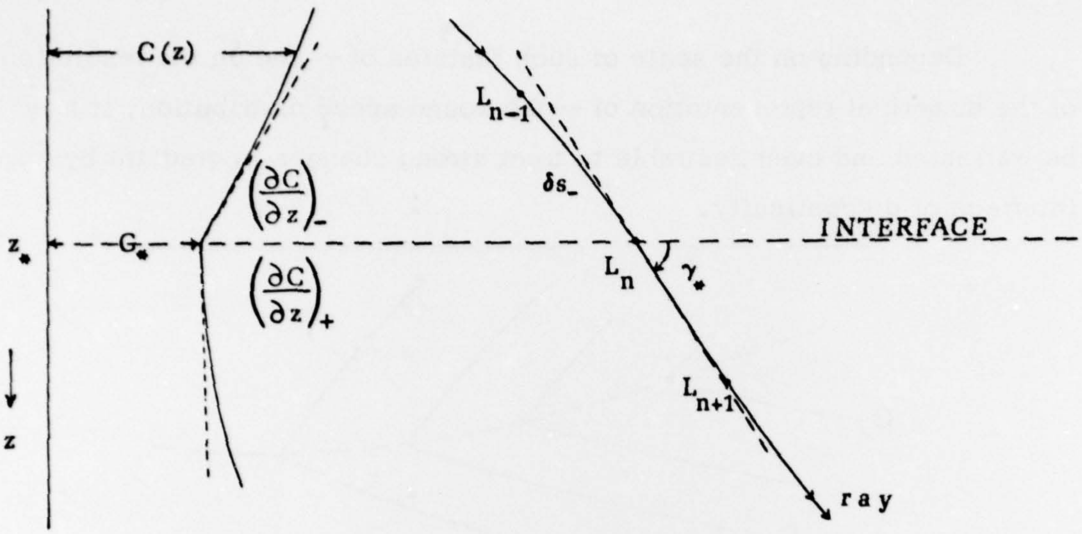


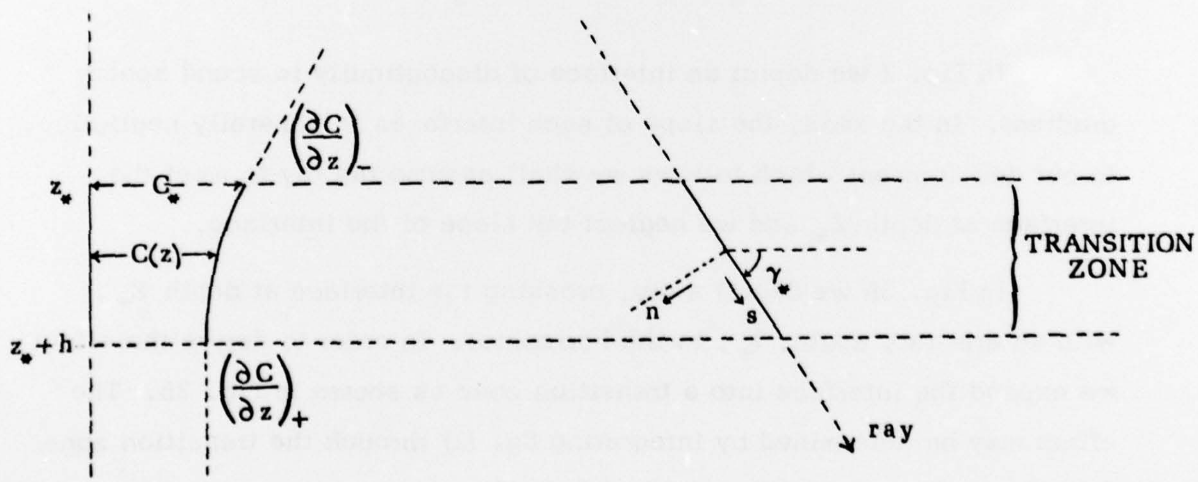
Fig. 2 An interface (dashed) of discontinuity in sound-speed gradient depicted by speed isolines.

In Fig. 2 we depict an interface of discontinuity in sound speed gradient. In the seas, the slope of such interfaces is generally negligible. In our development which follows we shall assume the ray to meet the interface at depth Z_* and we neglect the slope of the interface.

In Fig. 3A we depict a ray, crossing the interface at depth Z_*, with an arbitrary angle, γ_*, to the horizontal. In order to derive the effect we expand the interface into a transition zone as shown in Fig. 3B. The effect may be determined by integrating Eq. (2) through the transition zone, from Z_* to $Z_* + h$, and then letting h tend to zero.



3 A



3 B

Fig. 3 Discontinuity interface at Z_* (Fig. 3A), expanded into infinitesimal transition zone, with continuous gradient, of depth increment h (Fig. 3B).

The sound-speed profile, in the transition zone, is fitted by

$$C = C_* + \left[\frac{\partial C}{\partial Z}\right]_- (Z - Z_*) + \left[\left[\frac{\partial C}{\partial Z}\right]_+ - \left[\frac{\partial C}{\partial Z}\right]_-\right]\frac{(Z - Z_*)^2}{2\,h} \qquad (4)$$

which accomodates the value C_* at Z_*. Differentiation yields the gradient:

$$\frac{\partial C}{\partial Z} = \left[\frac{\partial C}{\partial Z}\right]_- + \left[\left[\frac{\partial C}{\partial Z}\right]_+ - \left[\frac{\partial C}{\partial Z}\right]_-\right]\frac{Z - Z_*}{h} \qquad (5)$$

which is continuous, going from the value just before the interface, $(\partial C/\partial Z)_-$, to that just after the interface, $(\partial C/\partial Z)_+$.

We express the integration of Eq. (2) through the transition zone by

$$\left[\frac{\partial L}{\partial s}\right]_+ = \left[\frac{\partial L}{\partial s}\right]_- - \int_{s_*}^{s_* + h/\sin\gamma_*} \frac{L}{C}\left[\frac{\partial^2 C}{\partial n^2} - \frac{1}{L}\,\frac{\partial C}{\partial s}\,\frac{\partial L}{\partial s}\right] ds \qquad (6)$$

Since we shall let h go to zero we anticipate the simplifications and reduce Eq. (6) to

$$\left[\frac{\partial L}{\partial s}\right]_+ = \left[\frac{\partial L}{\partial s}\right]_- - \frac{L_*}{C_*}\int_{s_*}^{s_* + h/\sin\gamma_*} \partial^2 C/\partial n^2 \quad ds \qquad (7)$$

For the transition zone we have

$$\frac{\partial^2 C}{\partial n^2} = \cos^2 \gamma_* \frac{\partial^2 C}{\partial Z^2} = \cos^2 \gamma_* \left[\left[\frac{\partial C}{\partial Z}\right]_+ - \left[\frac{\partial C}{\partial Z}\right]_- \right] \frac{1}{h} \tag{8}$$

which is independent of s. Thus Eq. (7) yields

$$\left[\frac{\partial L}{\partial s}\right]_+ = \left[\frac{\partial L}{\partial s}\right]_- - \frac{L_*}{C_*} \frac{\cos^2 \gamma_*}{\sin \gamma_*} \left[\left[\frac{\partial C}{\partial Z}\right]_+ - \left[\frac{\partial C}{\partial Z}\right]_- \right] \tag{9}$$

which is unchanged as $h \to 0$.

3. Formula for Reflections

For reflections we have the added complication that the sign of L is flipped:

$$L_R = - L_I \tag{10}$$

The subscript I is used to denote an incident value and the subscript R a reflected value. Taking this into account, the spreading rate changes according to

$$\left[\frac{\partial L}{\partial s}\right]_R = - \left[\frac{\partial L}{\partial s}\right]_I + \frac{2 L_R}{C} \frac{\cos^2 \gamma_I}{\sin \gamma_I} \frac{\partial C}{\partial Z} . \tag{11}$$

For numerical treatment we substitute

$$\left[\frac{\partial L}{\partial s}\right]_I \approx \frac{L_I - L_{I-1}}{\delta s} \tag{12}$$

$$\left[\frac{\partial L}{\partial s}\right]_R \approx \frac{L_R - L_{R-1}}{\delta s} \tag{13}$$

and Eq. (10) into Eq. (11) to obtain

$$L_{R-1} = - L_{I-1} - 2\frac{L_R}{C} \frac{\cos^2 \gamma_I}{\sin \gamma_I} \frac{\partial C}{\partial Z} \delta s \tag{14}$$

The new values, L_R and L_{R-1}, are used in place of L_I and L_{I-1} in proceeding with the integration of Eq. (2) along the ray following reflection.

Reference

[1] "The wavefront-divergence factor in ray-intensity integration", M. M. Holl, Meteorology International Incorporated, Project M-140; Technical Note Two, Contract No. N62271-67-M-2000, April 1967.